Using the integral based definition of $LN(x)$, prove that $LN(xy) = LN(x) + LN(y)$ using substitution. **SCORE: _____ / 15 PTS**
(Substitution was the technique used in lecture to the prove that $LN(\frac{x}{y}) = LN(x) - LN(y)$.)

$$LN(x) + LN(y) = \int_1^x \frac{1}{t}\,dt + \int_1^y \frac{1}{t}\,dt$$

LET $u = xt \longrightarrow t = \frac{u}{x}$

$du = x\,dt \longrightarrow dt = \frac{1}{x}du$

$t = 1 \longrightarrow u = x$

$t = y \longrightarrow u = xy$

$$\int_1^y \frac{1}{t}\,dt = \int_x^{xy} \frac{1}{\frac{u}{x}} \cdot \frac{1}{x}du$$

$$= \int_x^{xy} \frac{1}{u}\,du$$

$$= \int_x^{xy} \frac{1}{t}\,dt$$

SO $LN(x) + LN(y) = \int_1^x \frac{1}{t}\,dt + \int_x^{xy} \frac{1}{t}\,dt$

$$= \int_1^{xy} \frac{1}{t}\,dt$$

$$= LN(xy)$$

Find the length of the curve $y = \int_1^x \sqrt{(t+3)(t+5)}\,dt$ on $[2, 6]$. **SCORE: _____ / 25 PTS**

$$\frac{dy}{dx} = \sqrt{(t+3)(t+5)}$$

$$S = \int_2^6 \sqrt{1 + \sqrt{(t+3)(t+5)}^2}\,dt$$

$$= \int_2^6 \sqrt{1 + t^2 + 8t + 15}\,dt$$

$$= \int_2^6 \sqrt{t^2 + 8t + 16}\,dt$$

$$= \int_2^6 (t+4)\,dt$$

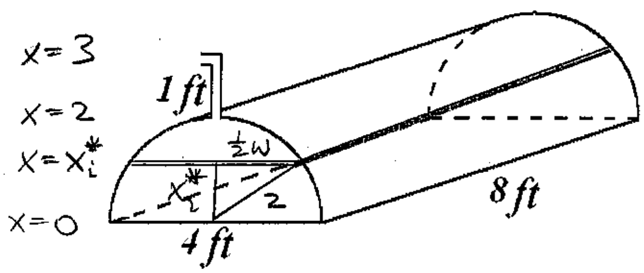
$$= \left(\tfrac{1}{2}t^2 + 4t\right)\Big|_2^6$$

$$= \tfrac{1}{2}(36-4) + 4(6-2)$$

$$= 16 + 16 = 32$$

The semi-cylindrical tank shown on the right is filled with water. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the work done in pumping all the water out through the spout. **SCORE: _____ / 20 PTS**

"SLICE" AT $x = x_i^*$



$d = (3 - x_i^*)$ FT

$F = \rho V$

$= \rho(2\sqrt{4-(x_i^*)^2})(8)\Delta x$ LB

$= 16\rho\sqrt{4-(x_i^*)^2}\Delta x$ LB

$(x_i^*)^2 + (\tfrac{1}{2}w)^2 = 2^2$

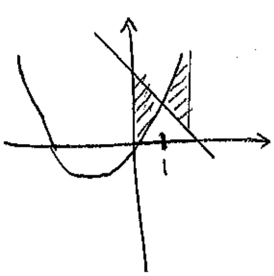
$\tfrac{1}{4}w^2 = 4 - (x_i^*)^2$

$w = 2\sqrt{4-(x_i^*)^2}$

TOTAL WORK $= \int_0^2 16\rho(3-x)\sqrt{4-x^2}\,dx$ FT-LB

Find the area between the curves $y = x^2 + 3x$ and $y = 5 - x$ on the interval $0 \le x \le 3$. **SCORE: _____ / 25 PTS**

Your final answer must be a number.



$x^2 + 3x = 5 - x$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = \cancel{-5}, 1$

$\int_0^1 (5 - x - (x^2+3x))\,dx + \int_1^3 (x^2+3x-(5-x))\,dx$

$= \int_0^1 (5 - 4x - x^2)\,dx + \int_1^3 (x^2 + 4x - 5)\,dx$

$= (5x - 2x^2 - \tfrac{1}{3}x^3)\Big|_0^1 + (\tfrac{1}{3}x^3 + 2x^2 - 5x)\Big|_1^3$

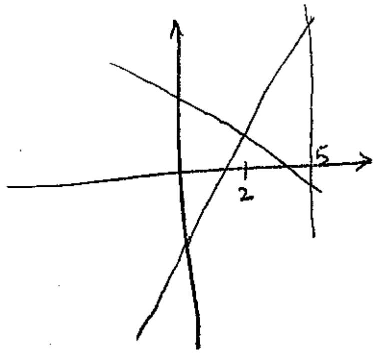
$= 5(1-0) - 2(1-0) - \tfrac{1}{3}(1-0)$
$+ \tfrac{1}{3}(27-1) + 2(9-1) - 5(3-1)$

$= 5 - 2 - \tfrac{1}{3} + \tfrac{26}{3} + 16 - 10$

$= 9 + \tfrac{25}{3}$

$= \tfrac{52}{3}$

The base of a solid is the region bounded by $y = 2x - 3$, $y = 3 - x$ and $x = 5$. Cross sections perpendicular to the $x -$ axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral for the volume of the solid. **SCORE: _____ / 20 PTS**



$2x - 3 = 3 - x$

$3x = 6$

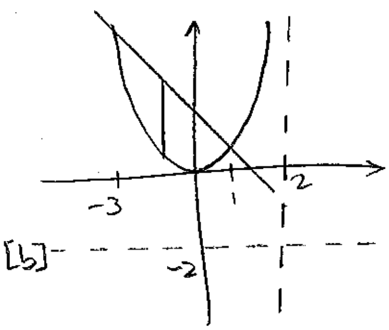
$x = 2$

$\frac{\pi}{8}\int_2^5 (2x-3-(3-x))^2\,dx$

$= \frac{\pi}{8}\int_2^5 (3x-6)^2\,dx$

Consider the region bounded by $y = x^2$ and $y = 3 - 2x$. **SCORE: _____ / 45 PTS**

[a] Find the volume of the solid if the region is revolved around the line $x = 2$. **<u>Your final answer must be a number.</u>**



$x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = 1, -3$

$\int_{-3}^{1} 2\pi(2-x)(3-2x-x^2)\,dx$

$= 2\pi\int_{-3}^{1}(6-4x-2x^2-3x+2x^2+x^3)\,dx$

$= 2\pi\int_{-3}^{1}(6-7x+x^3)\,dx$

$= 2\pi\left(6x - \frac{7}{2}x^2 + \frac{1}{4}x^4\right)\Big|_{-3}^{1}$

$= 2\pi\left(6(1+3) - \frac{7}{2}(1-9) + \frac{1}{4}(1-81)\right)$

$= 2\pi(24 + 28 - 20)$

$= 2\pi(32)$

$= 64\pi$

[b] Write, **BUT DO NOT EVALUATE**, a **<u>single</u>** integral for the volume of the solid if the region is revolved around the line $y = -2$.

$\pi\int_{-3}^{1}((3-2x- {}^-2)^2 - (x^2 - {}^-2)^2)\,dx$

$= \pi\int_{-3}^{1}((5-2x)^2 - (x^2+2)^2)\,dx$